

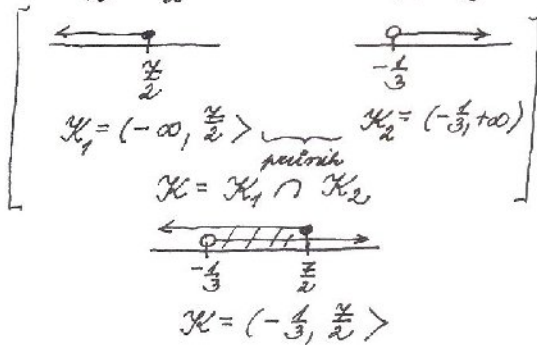
4. Soustavy lineárních nerovnic s jednou neznámou

Příklady

① Řeš v $\mathbb{R}$

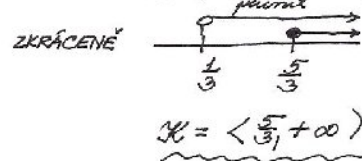
a) $2x - 7 \leq 0$ $\sigma = \mathbb{R}$
 $3x + 1 > 0$ $\sigma = \mathbb{R}$

$2x - 7 \leq 0$ a současně $3x + 1 > 0$
 $2x \leq 7$ $3x > -1$
 $x \leq \frac{7}{2}$ $x > -\frac{1}{3}$



b) $1 - 3x < 0$ $\sigma = \mathbb{R}$
 $3x - 2 \geq 5$ $\sigma = \mathbb{R}$

$1 - 3x < 0$ $\wedge$ $3x - 2 \geq 5$
 $-3x < -1$ $1 : (-3)$ $3x \geq 7$
 $x > \frac{1}{3}$ $x \geq \frac{7}{3}$

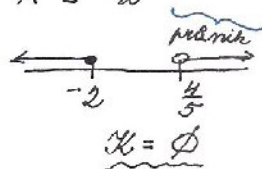


- VYŘEŠÍME KAŽDOU NEROVNICI ZVLÁŠT'
 - MNOŽINA VŠECH ŘEŠENÍ K

$K = K_1 \cap K_2$

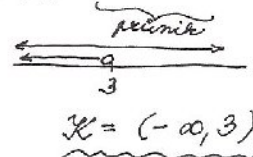
c) $2x + 3 \leq x + 1$ $\sigma = \mathbb{R}$
 $4x > 4 - x$ $\sigma = \mathbb{R}$

$2x + 3 \leq x + 1$ $\wedge$ $4x > 4 - x$
 $x \leq -2$ $5x > 4$ $1 : 5$
 $x > \frac{4}{5}$



d) $2x < x + 3$ $\wedge$ $x + 3 < x + 10$ $\sigma = \mathbb{R}, \sigma = \mathbb{R}$

$2x < x + 3$ $\wedge$ $x + 3 < x + 10$
 $x < 3$ $0x < 7$
 $0 < 7$ $\text{platí } K_2 = \mathbb{R}$



② Řeš v $\mathbb{Z}$

$\frac{x-1}{4} + \frac{7-x}{2} > 3$ $\wedge$ $2x-3 - \frac{x^2+2}{3} \leq 2 - \frac{(3-x)^2}{3}$ $\sigma = \mathbb{Z}$
 $\sigma = \mathbb{Z}$

$x-1 + 2(7-x) > 12$

$6x-9 - (x^2+2) \leq 6 - (3-x)^2$

$x-1 + 14-2x > 12$

$6x-9-x^2-2 \leq 6-(9-6x+x^2)$

$-x+13 > 12$ $1-13$

$-x^2+6x-11 \leq 6-9+6x-x^2$

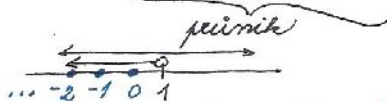
$-x > -1$ $1 \cdot (-1)$

$0x-11 \leq -3$ $1+11$

$! x < 1$

$0x \leq 8$

$(0 \leq 8)$ $\text{platí } K_2 = \mathbb{R}$



Výsledkem jako v $\mathbb{R} - K'$, pak
 přímka $\sigma = \mathbb{Z}$ - myslíme jen celá čísla

! ALE $\sigma = \mathbb{Z} \Rightarrow K = K' \cap \sigma = (-\infty, 1) \cap \mathbb{Z}$ (celá křiv.
 $K = \{0, -1, -2, \dots\} = \{x \in \mathbb{Z}; x < 1\} = \mathbb{Z}_0^-$ (celá křiv.
 $\{x \in \mathbb{Z}; x \leq 0\}$ NEKADNÁ CELÁ

③ Učít definici dle

a) $\sqrt{2x - \frac{2x-11}{4} - \frac{19-2x}{2}} + \sqrt{\frac{x}{3} + \frac{x-1}{3} - \frac{2x+15}{9}}$

VÝRAZY POD DRUHOU
ODHOCNINOU MUSÍ BÝT ≥ 0
(protože lát je definov.
pro $a \geq 0$)

Q: $2x - \frac{2x-11}{4} - \frac{19-2x}{2} \geq 0 \wedge \frac{x}{3} + \frac{x-1}{3} - \frac{2x+15}{9} \geq 0$

$\sigma = \mathbb{R}$

$8x - (2x-11) - 2(19-2x) \geq 0 \quad 3x + 3(x-1) - (2x+15) \geq 0$

$8x - 2x + 11 - 38 + 4x \geq 0 \quad 3x + 3x - 3 - 2x - 15 \geq 0$

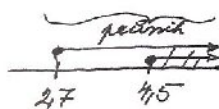
$10x - 27 \geq 0 \quad 4x - 18 \geq 0$

$10x \geq 27 \quad 4x \geq 18$

$x \geq \frac{27}{10} \quad x \geq \frac{18}{4}$

$x \geq 2,7 \quad x \geq \frac{9}{2}$

$x \geq 4,5$



POZOR - URČUJEME
def. obor Q

$Q = \langle 4,5; +\infty) \cap \left[\frac{9}{2}, +\infty) \right]$

b) $\sqrt{\frac{3-11x}{4} - 6x+8} + \frac{1}{\sqrt{(x+2)^2 - x(x+2)}}$ $\sigma = \mathbb{R}$

Q: $\frac{3-11x}{4} - 6x+8 \geq 0 \wedge (x+2)^2 - x(x+2) \geq 0 \wedge (x+2)^2 - x(x+2) \neq 0$

$3-11x-24x+32 \geq 0 \quad (x+2)^2 - x(x+2) > 0$

$-35x+35 \geq 0 \quad x^2+4x+4-x^2-2x > 0$

$-35x \geq -35 \quad 2x+4 > 0$

$! \quad 35x \leq 35 \quad 2x > -4$

$x \leq 1 \quad x > -2$



$Q = (-2, 1)$

c) $\sqrt{\frac{11-4x}{5} - 4x-7} + \frac{1}{\sqrt{(x+3)^2 - x(x-3)}}$ $\sigma = \mathbb{R}$

Q: $\frac{11-4x}{5} - 4x-7 \geq 0 \wedge (x+3)^2 - x(x-3) \geq 0 \wedge (x+3)^2 - x(x-3) \neq 0$

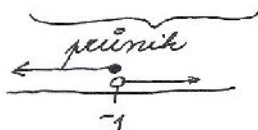
$11-4x-20x-35 \geq 0 \quad (x+3)^2 - x(x-3) > 0$

$-24x-24 \geq 0 \quad x^2+6x+9-x^2+3x > 0$

$-24x \geq 24 \quad 9x+9 > 0$

$! \quad 24x \leq -24 \quad 9x > -9$

$x \leq -1 \quad x > -1$



$Q = \emptyset$

④ Řeš rovnici kvadratické a nerovnici v $\mathbb{R}$

$$x-2 = \frac{2-3x}{8} + \frac{8-x}{12} \quad | \cdot 24 \quad \wedge \quad x^2+2x-1 > (x-2)^2-3 \quad \sigma = \mathbb{R}$$

$$24x-48 = 3(2-3x) + 2(8-x) \quad \wedge \quad x^2+2x-1 > x^2-4x+4-3$$

$$24x-48 = 6-9x+16-2x$$

$$2x-1 > -4x+1 \quad | +4x+1$$

$$24x-48 = 22-11x \quad | +11x+48$$

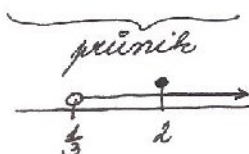
$$6x > 2 \quad | :6$$

$$35x = 70$$

$$x > \frac{2}{6}$$

$$x = 2$$

$$x > \frac{1}{3}$$



$$\mathcal{K} = \{2\}$$

⑤ Řeš v $\mathbb{R}$

$$a) \quad \frac{1}{3}x+1 < 3x \quad \wedge \quad 5-2x \leq 4x-1 \quad \wedge \quad 2+x > \frac{-x-2}{4} \quad \sigma = \mathbb{R}$$

$$x+3 < 9x$$

$$-6x \leq -6$$

$$8+4x > -x-2$$

$$3 < 8x$$

$$! \quad 6x \geq 6$$

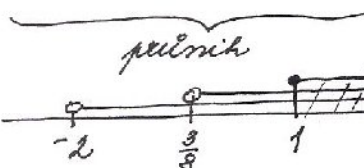
$$5x > -10$$

$$8x > 3 \quad | :8$$

$$x \geq 1$$

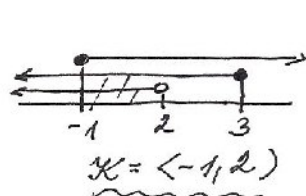
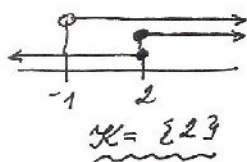
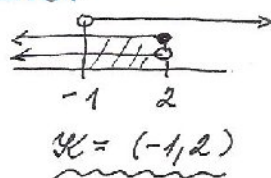
$$x > -2$$

$$x > \frac{3}{8}$$



$$\mathcal{K} = \langle \frac{3}{8}, 1 \rangle$$

KDYBY



$\mathcal{K}$ b) $-3 \leq x-2 < 2x+1 < \frac{1}{2}x \quad \sigma = \mathbb{R}$

$$-3 \leq x-2 \quad \wedge \quad x-2 < 2x+1 \quad \wedge \quad 2x+1 < \frac{1}{2}x \quad | \cdot 2$$

$$-1 \leq x$$

$$-3 < x$$

$$4x+2 < x$$

$$x \geq -1$$

$$x > -3$$

$$3x < -2$$

$$x < -\frac{2}{3}$$



$$\mathcal{K} = \langle -1, -\frac{2}{3} \rangle$$